

4. 二元关系

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P90

2.

解:

$$(1) xy \geq 0$$

$$(2) x \geq y$$

$$(3) \emptyset$$

3.

解:

$$R_1 \circ R_2 = \{(c, d)\}$$

$$R_2 \circ R_1 = \{(a, c), (a, d)\}$$

$$R_1^2 = \{(a, a), (a, b), (a, d)\}$$

$$R_2^2 = \{(b, b), (c, c), (c, d)\}$$

$$R_2^3 = \{(b, c), (b, d), (c, b)\}$$

4.

证明:

$$\text{任取 } (a, c) \in R_1 \circ (R_2 \cap R_3)$$

$$\text{则存在 } (a, b) \in (R_2 \cap R_3), (b, c) \in R_1$$

$$\therefore (a, b) \in R_2 \text{ 且 } (a, b) \in R_3$$

$$\therefore (a, c) \in R_1 \circ R_2 \text{ 且 } (a, c) \in R_1 \circ R_3$$

$$\therefore (a, c) \in ((R_1 \circ R_2) \cap (R_1 \circ R_3))$$

$$\therefore R_1 \circ (R_2 \cap R_3) \subseteq (R_1 \circ R_2) \cap (R_1 \circ R_3)$$

7.

证明:

$$|X| \models |X| \therefore \text{自反}$$

$$|X| \models |Y| \Leftrightarrow |Y| \models |X| \therefore \text{对称}$$

$$|X| \models |Y|, |Y| \models |Z| \Rightarrow |X| \models |Z| \therefore \text{传递}$$

$$\therefore \sim \text{是 } P(A) \text{ 上的等价关系}$$

$$\text{商集 } P(A) / \sim = \{\emptyset, \{1\}, \{1, 2\}, \{1, 2, 3\}, \{1, 2, 3, 4\}\}$$

10.

证明:

$$\forall a \in A,$$

$$(a, a) \in R, (a, a) \in A \times A$$

$$\therefore (a, a) \in R \cap (A \times A)$$

\therefore 自反

$$\forall (a, b), (b, a) \in R \cap (A \times A)$$

$$(a, b), (b, a) \in R$$

$$\therefore a=b$$

\therefore 反对称

$$\forall (a, b), (b, c) \in R \cap (A \times A)$$

$$(a, b), (b, c) \in R$$

$$\therefore (a, c) \in R$$

$$\text{又 } (a, c) \in A \times A$$

$$\therefore (a, c) \in R \cap (A \times A)$$

\therefore 传递

$\therefore R \cap (A \times A)$ 是 A 上的一个部分序。

11.

证明:

$$\forall R_1 \in B$$

$$xR_1y \Rightarrow xR_1y$$

$$\therefore R_1 \leq R_1$$

\therefore 自反

$$\forall R_1 \leq R_2 \text{ 且 } R_2 \leq R_1$$

$$R_1 \subseteq R_2 \text{ 且 } R_2 \subseteq R_1$$

$$\therefore R_1 = R_2$$

\therefore 反对称

$$\forall R_1 \leq R_2 \text{ 且 } R_2 \leq R_3$$

$$\text{则 } R_1 \subseteq R_2 \text{ 且 } R_2 \subseteq R_3$$

$$\therefore R_1 \subseteq R_3$$

$$\therefore R_1 \leq R_3$$

\therefore 传递

$\therefore \langle B, \leq \rangle$ 是部分序集。